

INELASTIC CONSTITUTIVE PARAMETER IDENTIFICATION USING AN EVOLUTIONARY ALGORITHM WITH CONTINUOUS INDIVIDUALS*

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SUMMARY

This paper presents a method for identifying the parameter set of inelastic constitutive equations, which is based on an evolutionary algorithm proposed by the authors. The advantage of the method is that appropriate parameters can be identified even when the measured data are subject to considerable errors and the model equations are inaccurate. The design of experiments suited for the parameter identification of a material model by Chaboche under the uniaxial loading and stationary temperature conditions was first considered. Then the parameter set of the model was identified by the proposed method from a set of experimental data. In comparison to those by other methods, the resultant stress–strain curves by the proposed method correlated well and reliably to the actual material behaviors.

KEY WORDS: inelasticity; parameter identification; evolutionary algorithms; continuous search space

1. INTRODUCTION

Up to now, there has been an accelerating rate at which various structural materials have been developed to assist the objective of industrial designers. In many industrial fields, materials are often used under severe operating conditions such as cyclic loading, high temperature, high pressure and high irradiation, for example, if they are used as pressure vessels and pipes of a nuclear plant. For the reliable evaluation of deformation behaviours of these materials, thermo-inelastic analysis is indispensable. A variety of theoretical models have been proposed and discussed in the referenced literature to describe a wide range of viscoplastic behaviours of metallic materials.^{1–7} Viscoplastic constitutive equations derived from these theories involve many parameters, which significantly influence the behaviours of the constitutive equations. Appropriate parameters must be determined accordingly such that the accurate behaviours of materials can be expressed.

Every constitutive equation has its own method for the parameter identification. In conventional approaches, the model of interest is first approximated and its parameters are identified sequentially through the curve fitting approach.⁸ However, the determination of its process is problem-dependent, and thus may not be easy if the model is highly non-linear. In addition, the process may yield significant errors due to the model approximation, particularly when the parameter space is high-dimensional.

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On the other hand, the advance of computer hardware has increased the popularity of an approach where all the parameters are identified simultaneously⁹ and, most commonly, optimization methods are used to find the parameter set by adjusting them until they provide the best agreement between the measured data and the computed model response. As a result, a number of calculus-based techniques were proposed and incorporated to solve this optimization problem.¹⁰⁻¹² These techniques, however, can fail in the actual situation, for example, when the measured data are noisy or the model equations are inaccurate, since they can cause the objective function to be complicated.

Two major strategies to deal with this straightforward problem may be to

- (a) modify the objective function by a regularization or by choosing different weighting factors, and
- (b) use a different optimization method so that the optimization cannot fail.

Most researchers' efforts were put on approach (a),¹³⁻¹⁷ but industrial engineers often come across the difficulty that it is hard to determine parameters added in the objective function.

On the other hand, Evolutionary Algorithms (EAs), which have been named to represent Genetic Algorithms (GAs),¹⁸ Evolutionary Programming (EP),¹⁹ Evolution Strategies (ESs)^{20,21} and their recombined algorithms,²² have appeared as robust optimization techniques in the last few decades. EAs are based on the collective learning process within a population of individuals, each of which represents a search point in the space of potential solutions to a given problem. Each of these algorithms has clearly demonstrated its capability to yield good approximate solutions even in the case of complicated multimodal, discontinuous, non-differentiable, and even noisy or moving response surface optimization problems, and has been successfully implemented in areas of structural design, control, etc.

In this paper, we therefore propose to use EAs for identifying the parameter set of inelastic constitutive equations in accordance to approach (b). The advantage of the proposed method is that parameters can be identified without any divergence in every case. Furthermore, an EA, which can search for a potential solution efficiently, is proposed in this paper. The algorithm is based on GAs, which have been most commonly used due to their promising performance for a broad range of optimization problems, but its individual representation and reproductive processes are formulated from a different viewpoint to achieve steady and fast convergence.

In the next section, the inelastic constitutive equation is described in a general form and Chaboche's model⁴ is introduced as an example for an inelastic material law. The third section deals with the overview of the non-linear parameter identification and the fundamentals of EAs, as well as the proposed formulation using EAs. The fourth section refers to the EA proposed by the authors, and its comparison with GAs is further presented. In the fifth section, the parameter identification of Chaboche's model is formulated in accordance to EAs and the results of numerical examples are described. Conclusions of the paper are summarized in the final section.

2. INELASTIC CONSTITUTIVE EQUATIONS

In general, constitutive relations are given in state space form for the strain ε and a set of ζ internal variables $\xi \in \mathbb{R}^{\zeta}$ and, typically in the frame of the small strain approximation, have the following form:

$$\dot{\varepsilon} = \hat{\varepsilon}(\varepsilon, \xi, \theta, \sigma, \kappa) \quad (1a)$$

$$\dot{\xi} = \hat{\xi}(\varepsilon, \xi, \theta, \sigma, \kappa) \quad (1b)$$

where θ and σ are the temperature and stress, respectively, and $\kappa \in \mathbb{R}^r$ represents a vector of r material parameters. The following initial conditions are given for their direct analysis:

$$\varepsilon|_{t=0} = \varepsilon_0 \tag{1c}$$

$$\xi|_{t=0} = \xi_0 \tag{1d}$$

Chaboche’s viscoplastic model, for instance, is capable of describing cyclic hardening and softening behaviours with the yielding surface and appears to model a wide range of inelastic material behaviour characteristics. Its formation under the uniaxial loading and stationary temperature conditions is given by

$$\sigma = E\varepsilon^e \tag{2a}$$

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \tag{2b}$$

$$\dot{\varepsilon}^p = \left\langle \frac{|\sigma - Y| - R}{K} \right\rangle^n \text{sgn}(\sigma - Y) \tag{2c}$$

$$\dot{Y} = H\dot{\varepsilon}^p - DY|\dot{\varepsilon}^p| \tag{2d}$$

$$\dot{R} = h|\dot{\varepsilon}^p| - dR|\dot{\varepsilon}^p| \tag{2e}$$

where variables σ , ε^e , ε^p , Y and R are the uniaxial stress, the uniaxial elastic strain, the uniaxial inelastic strain, the uniaxial back stress and the isotropic hardening variable, respectively, and the vector $\mathbf{\kappa} = [K, n, H, D, h, d, E]$ represents the material parameters. The notation $\langle \cdot \rangle$ in equation (2c) is zero if the value inside is negative. Initial conditions for the direct analysis of the model are given by

$$\varepsilon|_{t=0} = \varepsilon_0 \tag{2f}$$

$$Y|_{t=0} = Y_0 \tag{2g}$$

$$R|_{t=0} = R_0 \tag{2h}$$

In the cyclic loading test no external force is provided initially ($\varepsilon|_{t=0} = 0$, $Y|_{t=0} = 0$), and, thus, considering the fact that Young’s modulus E can be easily found from experiments, parameters K , n , H , D , h , d and R_0 must be determined to describe the performance of a specific material.

For computer simulation, a time integration algorithm such as the mid-point rule is used to discretize the equations.

3. NON-LINEAR PARAMETER IDENTIFICATION

3.1. Formulation

The parameter identification is generally defined as identifying the parameter set (vector) $\mathbf{x}^* \in X$ when the measured data (vector) $\mathbf{y}^* \in \Psi$ and direct mapping $\psi: X \rightarrow \Psi$ are known. The parameter space is given by

$$X = \{ \mathbf{x} \in X_1 \times \dots \times X_n \mid g_j(\mathbf{x}) \geq 0, \forall j \in \{1, \dots, q\} \} \tag{3}$$

where $g_i: X_1 \times \dots \times X_n \rightarrow \mathbb{R}$ represents inequality constraints. For the unconstrained problem, $X = X_1 \times \dots \times X_n$. Problems with the non-linear direct mapping Ψ are termed non-linear parameter identification problems.

In practice, deterministic models describe reality only in an idealized sense, and thus we may express the input–output relation as follows:

$$\mathbf{y} = \Psi(\mathbf{x}) + \mathbf{e} \tag{4}$$

where the error term $\mathbf{e} \in \Psi$ is given by

$$\mathbf{e} = \mathbf{e}_1 + \mathbf{e}_2 \tag{5}$$

Here \mathbf{e}_1 and \mathbf{e}_2 are errors in the measurement of \mathbf{y} and those in the model equations, respectively. In the analysis of field quantities shown in Figure 1, the model equations in general take the form

$$L(\kappa)\phi = q \tag{6}$$

where $L(\cdot)$, κ , ϕ and q are the differential operator, material property, field quantity and a force or source term, respectively.

Parameter identification problems for equation (6) can be classified into the following types in terms of the parameter set to be identified:²⁴

- (a) domain Ω , its unknown outer boundary or unknown inner boundary,
- (b) governing equations,
- (c) boundary conditions on the entire or partial boundary and/or the initial conditions,
- (d) force or source q applying in Ω ,
- (e) material properties κ defined in Ω and involved in the governing equations.

It is also possible that the inverse problem consists of more than one classification. In these problems, the input vector resides in the continuous space ($X \subseteq \mathbb{R}^n, n \in \mathbb{N}$), and thus we confine ourselves to continuous search space problems.

In reality, to be used as a method for an inverse problem whose solution is unique, the method must at least possess the following features:²⁴

- (i) The method can find a parameter set almost identical to the exact solution when the measured data and model equations are not subject to errors, i.e. when pseudo-experimental data created from computer simulations are provided.
- (ii) The method can still find a good approximate solution when the measured data and model equations are subject to errors, i.e. when the actual experimental data were used.

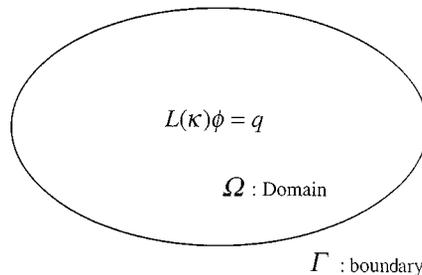


Figure 1. Problems of field quantities

Out of a number of strategies²⁵⁻³¹ that have been developed for solving inverse problems, minimizing a least square criterion has been most widely used. In this approach, optimization techniques are used to find \mathbf{x}^* by adjusting them until the measured data match the corresponding data computed from the parameter set in the least square fashion, i.e.

$$\min_{\mathbf{x}} f(\mathbf{x}) \tag{7a}$$

with the cost functional

$$f(\mathbf{x}) = \sum_{i=1}^m k_i \|y_i^* - \psi_i(\mathbf{x})\|^2 \tag{7b}$$

where $k_i > 0$ is a weighting factor, without any prior knowledge, normally being all set to one. At least some of its popularity is due to the fact that it can be applied in an *ad hoc* manner directly to the deterministic model without any cognizance being taken of the probability distribution of the observations.³² It is thus usually adequate for problems where the coefficients in the model equations have no physical significance.

Although various calculus-based optimization techniques have been intensively used to solve this optimization problem, these techniques may not be able to search properly if errors contained in the model equations and in the measurement cause the objective function to be severely non-linear such as non-convex or multimodal,³³ in the case of non-linear $\psi_i(\mathbf{x})$. This often occurs if the number of available measurement data is limited and, in such cases, the solution may vibrate, diverge, or result in a local minimum.

Therefore in sanction with the objective of the paper described in Section 1, we hereby propose to use EAs, which are significantly promising for complex optimization, and the next section presents the fundamentals of EAs.

3.2. Evolutionary algorithms

EAs are probabilistic optimization algorithms based on the model of natural evolution, and the algorithms have clearly demonstrated their capability to create good approximate solutions in complex optimization problems. The popularity of the algorithms is due to the following characteristics:

- (i) less possibility to converge to a local minimum as the search starts from a number of points;
- (ii) compatibility with the parallel computer;
- (iii) robustness since only objective function information is required;
- (iv) capability to find a solution in broad search space effectively through probabilistic operations.

Figure 2 shows the fundamental structure of EAs. First, a population of individuals, each represented by a vector, is initially (generation $t = 0$) generated at random, i.e.

$$P^0 = \{\mathbf{u}_1^0, \dots, \mathbf{u}_\lambda^0\} \in I^\lambda \tag{8}$$

where $\lambda \in \mathbb{N}$ and I represent the population size of parental individuals and the space of individual, respectively. The population then evolves towards better regions of the search space by means of randomized processes of recombination, mutation and selection though either the recombination or mutation operator is not implemented in some algorithms. In the recombination operator $\mathbf{r}: I^\lambda \rightarrow I^\gamma$, λ parental individuals based $\gamma (\in \mathbb{N})$ offspring individuals by combining

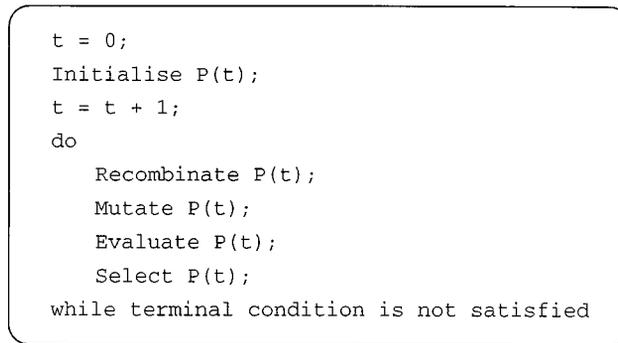


Figure 2. Fundamental structure of evolutionary algorithms

part of the information from the parental individuals. The mutation $\mathbf{m}: I^{\gamma} \rightarrow I^{\gamma}$ then forms new individuals by making large alterations with small possibility to the offspring individuals regardless of their inherent information. With the evaluation of fitness for all the individuals, the selection operator $\mathbf{s}: I^{\gamma} \cup I^{\gamma+\lambda} \rightarrow I^{\lambda}$ favourably selects individuals of higher fitness to produce more often than those of lower fitness. These reproductive operations form one generation of the evolutionary process, which corresponds to one iteration in the algorithm, and the iteration is repeated until a given terminal criterion is satisfied.

Out of the algorithms, GAs are most popular by the fact that their reproductive processes use only a couple of deterministic rules (mostly randomized processes), causing them to be applicable to a variety of complex optimization problems.

4. EVOLUTIONARY ALGORITHMS FOR REAL SEARCH SPACE

4.1. Formulation

EAs, in general, are rather formulated to comprise a broad range of optimization problems although the problems of interest, as described in Section 2, are characterized only with continuous search space. In this section we present the EA proposed by the authors, which was specifically formulated for the optimization with continuous search space. The reproductive operations of the proposed algorithm are intended to be similar to those of GAs such that it can take the advantage of probabilistic features in GAs. The major difference of the proposed algorithm from GAs is that the representation of the individual is given by a search point itself, i.e. a real continuous vector. This formulation was made with an assumption that the direct use of the search point may search more efficiently than the representation decoded into a binary string as used in GAs. In this case, the population at generation t is given by

$$P^t = \{\mathbf{x}_1^t, \dots, \mathbf{x}_\lambda^t\} \in X^\lambda \quad (9)$$

This representation makes up grasp the concept of the individual in a different manner. While the binary string in GAs represents a DNA chromosome, a microscopic or genetic representation of human being, the continuous vector representation corresponds to a set of macroscopic or phenomenological information of the human being.

The definition of the recombination and mutation becomes the probabilistic distribution of the phenomenological measures accordingly. If the two offspring individuals are formulated to be

created from a pair of randomly selected parental individuals as in GAs, $\mathbf{r}'' : I^2 \rightarrow I$, the recombination operation may be defined as

$$\begin{aligned} \mathbf{r}''(\mathbf{x}_\alpha^t, \mathbf{x}_\beta^t) &= (1 - \mu_\alpha^t) \cdot \mathbf{x}_\alpha^t + \mu_\beta^t \cdot \mathbf{x}_\beta^t \\ \mathbf{r}''(\mathbf{x}_\beta^t, \mathbf{x}_\alpha^t) &= \mu_\alpha^t \cdot \mathbf{x}_\alpha^t + (1 - \mu_\beta^t) \cdot \mathbf{x}_\beta^t \end{aligned} \tag{10}$$

where \mathbf{x}_α^t and \mathbf{x}_β^t are parental individuals at generation t and the coefficient $\mu_i^t, \forall i \in \{\alpha, \beta\}$, is defined by the normal distribution with mean 0 and standard deviation η_i^t :

$$\mu_i^t = N(0, \eta_i^{t^2}) \tag{11}$$

The standard deviation can adopt a self-adaptive strategy (variable with respect to t)³⁴ or be simply constant. The self-adaptive strategy makes the convergence rate required for each generation faster at the expense of the computation time and vice versa. Note that the mutation is not embedded in this operator since it can allow individuals to alter largely with small possibility, when the coefficient μ_i^t is large.

The evaluation of the fitness can be conducted with a linear scaling, where the fitness of each individual is calculated as the worst individual of the population subtracted from its objective function value:

$$\Phi(\mathbf{x}_i^t) = \max\{f(\mathbf{x}^t) | \mathbf{x}^t \in P^t\} - f(\mathbf{x}_i^t), \quad \forall i \in \{1, \dots, \lambda\} \tag{12}$$

as in GAs. $\Phi(\mathbf{x}_i^t) \geq 0$ is thus satisfied by this equation. Proportional selection,³⁵ which is the most popular selection operation in GAs, can also be directly used in the proposed algorithm as it requires $\Phi(\mathbf{x}_i^t) \geq 0$. In this selection, the reproduction probabilities of individuals $p_s : X \rightarrow [0, 1]$ are given by their relative fitness,

$$p_s(\mathbf{x}_i^t) = \frac{\Phi(\mathbf{x}_i^t)}{\sum_{j=1}^{\lambda} \Phi(\mathbf{x}_j^t)} \geq 0 \tag{13}$$

Optionally, ranking selection³⁶ can be implemented in the algorithm.

The advantage of the proposed algorithm is its simple formulation as well as the compatibility with GENESIS,³⁷ which is one of the most popular GA software (only 200 lines out of 2400 lines were modified). This enables us to use most of the options available for GENESIS. It is, therefore, also possible to compare its performance with the GA directly, and the next section will discuss the comparison from several performance tests.

4.2. Comparison

4.2.1. Test functions. As it is impossible to predict the behaviour of the algorithms by theoretical considerations, a set of test functions having continuous search space were prepared to demonstrate the capability of both the canonical GA, which is composed of the one-point crossover³⁸ and proportional selection, and the proposed algorithm. The mathematical characteristics of the test functions are unimodal/multimodal, quadratic/non-quadratic, convex/non-convex and continuous/discontinuous. The test functions are as follows:

$$\begin{aligned} \text{Func. I: } f_1(\mathbf{x}) &= \sum_{i=1}^n x_i^2, \quad \mathbf{x} \in \mathbb{R}^n; \quad n = 30 \\ &- 5.12 \leq x_i \leq 5.12, \quad \mathbf{x}^* = [0, \dots, 0]^T \\ f_1(\mathbf{x}^*) &= 0 \end{aligned} \tag{14}$$

$$\begin{aligned} \text{Func. II: } f_2(\mathbf{x}) &= 6n + \sum_{i=1}^n \lfloor x_i \rfloor, \quad \mathbf{x} \in \mathbb{R}^n; \quad n = 5 \\ -5 \cdot 12 &\leq x_i \leq 5 \cdot 12, \quad \mathbf{x}^* = [-5 \cdot 12, \dots, -5]^T \\ f_2(\mathbf{x}^*) &= 0 \end{aligned} \quad (15)$$

$$\begin{aligned} \text{Func. III: } f_3(\mathbf{x}) &= nA + \sum_{i=1}^n x_i^2 - A \cos(\omega x_i), \quad \mathbf{x} \in \mathbb{R}^n; \quad n = 20, \quad A = 10, \quad \omega = 2\pi \\ -5 \cdot 12 &\leq x_i \leq 5 \cdot 12, \quad \mathbf{x}^* = [0, \dots, 0]^T \\ f_3(\mathbf{x}^*) &= 0 \end{aligned} \quad (16)$$

Func. IV: Func. III except that $\omega = \pi/2$.

Func. I is the simplest quadratic function, which is also characterized by unimodality, continuity and convexity. It is often used as the first test case of non-linear functions since many objective functions formulated in reality take this form. Func. II, introduced by De Jong,³⁹ is a simple linear but discontinuous function, which comprises a number of plateaus by the integer operator $\lfloor \cdot \rfloor$. All gradient-based methods are not useful for this function due to its discontinuity. The discontinuity gives difficulty even to other optimization methods since there is no guidance towards the edge of each plateau. Funcs. III and IV are continuous, multimodal test functions, which are generated by modifying the value of parameter ω . The last term of the function yields a number of local optima, which renders optimization methods difficult to search, depending on the initial search point. Three-dimensional graphical representations of these functions are shown in Figures 3–6.

4.2.2. Search performances. Internal parameters selected for both the algorithms are listed in Table I. The crossover and mutation rates in the canonical GA, 0.6 and 0.0001, are typically used in many publications. The standard deviation of the proposed algorithm was set to be constant for simplicity. All the other internal parameters were set identical for both the algorithms. The reader is referred to Reference 37 for the definition of the parameters which are not explained in this paper. Note here that the elitist strategy³⁵ was incorporated in the selection process of both

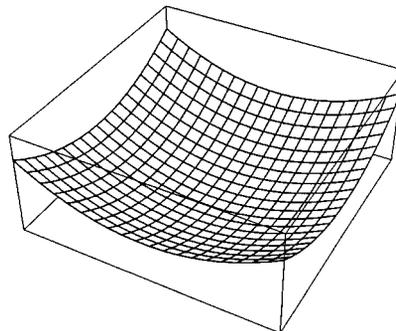


Figure 3. 3-D representation of Func. I

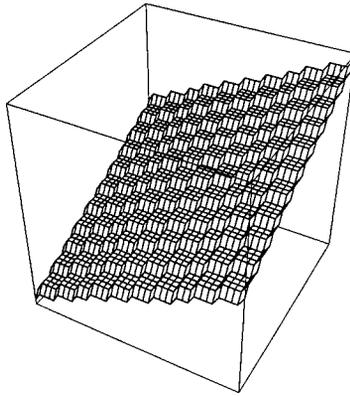


Figure 4. 3-D representation of Func. II

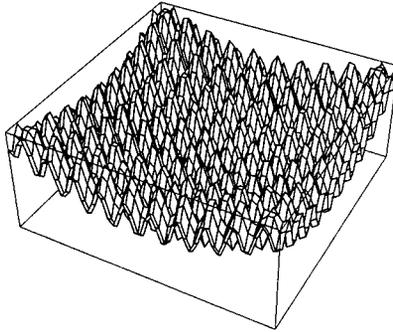


Figure 5. 3-D representation of Func. III

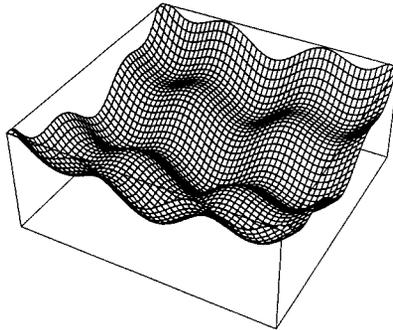


Figure 6. 3-D representation of Func. IV

the algorithms. For generality, ten runs were performed for each test, and the average performance of each algorithm was taken.

The results of the search performance of both the algorithms until 2500 generations are shown in Figures 7–10. In the figures, real lines indicate the outcome from the proposed method whereas

Table I. Internal parameters for both the algorithms

	Canonical GA	Proposed algorithm
Population size	50	50
Bit length per variable	30	
Crossover rate	0.95	
Mutation rate	0.001	
Standard deviation		0.5 (constant)
Generation gap	5	5
Scaling window	1.0	1.0

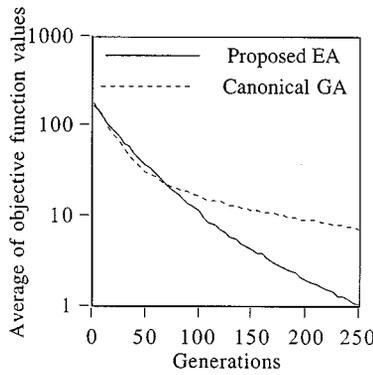


Figure 7. Average of objective function values vs. generations for Func. I

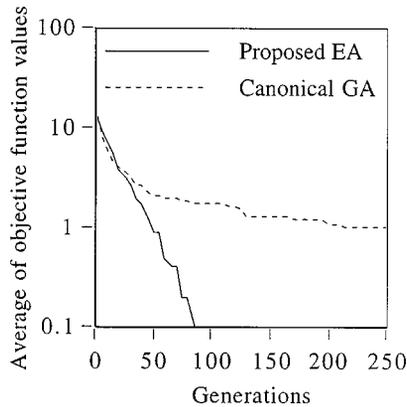


Figure 8. Average of objective function values vs. generations for Func. II

the results by the canonical GA are indicated by the broken lines. The result of the first test clearly reflects the superiority of the proposed algorithm on the unimodal function optimization. The second result then indicates that the proposed algorithm still outperforms the canonical GA even when the function is partially discontinuous. The canonical GA, in fact, could not find the global

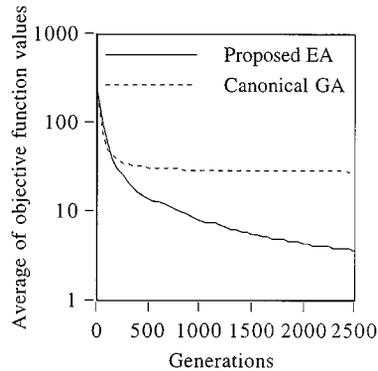


Figure 9. Average of objective function values vs. generations for Func. III

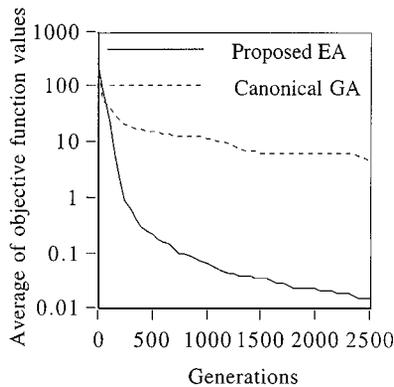


Figure 10. Average of objective function values vs. generations for Func. IV

optimum $\mathbf{x}^* = \mathbf{0}$ in all the ten runs within the prescribed number of generations whereas the proposed algorithm resulted in the global optimum in all the cases. The proposed algorithm also has a better performance of Funcs. III and IV than GA. The GA was nearly in the state of 'premature convergence', having a slow convergence rate before reaching the global optimum, for both the tests.

4.2.3. Computation time and memory usage. In this section, the computation time and memory required for the optimization of Func. I up to 2000 generations with Sun SPARKStation II were compared for both the algorithms. Parameters listed in Table I were used, except the bit length per variable for GA, which was varied every five bits within a range of 10 and 30.

The results of the computation time and memory usage are shown in Figures 11 and 12. These results indicate that, for the canonical GA, the more the computation time and memory usage, the longer the bit length per variable. The fact that the computation time increases with the precision of each variable increased is obviously caused by the additional computational effort for decoding. On the other hand, the reason for the increase of the memory usage is that both the binary and decoded representations must be saved in memory. Although the 30-bit representation was far beyond the real representation in precision, the GA required seven times of computation time and eight times of memory usage of the proposed algorithm.

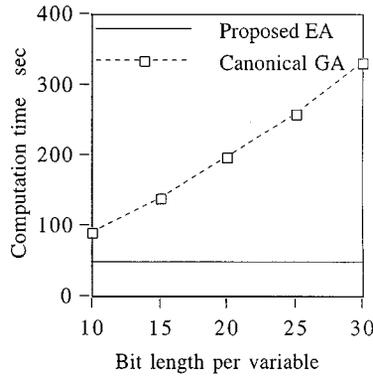


Figure 11. Computation time vs. bit length

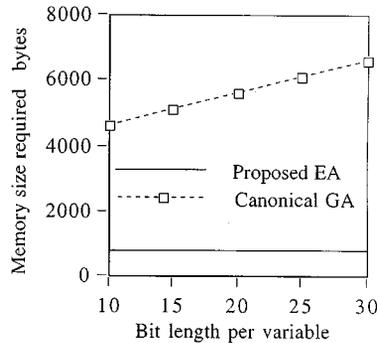


Figure 12. Memory usage vs. bit length

Conclusively, the proposed algorithm is superior to GAs in all the performance, computation time and memory usage for continuous search space problems. The next section will deal with the parameter identification of inelastic constitutive equations using the proposed algorithm.

5. PARAMETER IDENTIFICATION OF INELASTIC CONSTITUTIVE EQUATIONS

5.1. Formulation for parameter identification of Chaboche's model

There were seven parameters to be determined for Chaboche's model described in Section 2. Let the parameter set $\mathbf{x}^T = [K, n, H, D, h, d, R_0]$, and represent the constitutive equations (2) with strain ε as the input variable with respect to time and stress σ as the output variable in the following form:

$$\sigma = \psi(\mathbf{x}, \varepsilon) \tag{17}$$

where $\psi : \mathbb{R}^7 \times \mathbb{R} \rightarrow \mathbb{R}$. If m pairs of stress–strain data $\{[\sigma_1^*, \epsilon_1^*], \dots, [\sigma_m^*, \epsilon_m^*]\}$ are used to determine the parameter set, then the optimization problem to be formulated according to Section 3 is

$$\min_{\mathbf{x}} \sum_{i=1}^m k_i \|\sigma_i^* - \psi(\mathbf{x}, \epsilon_i^*)\|^2 \tag{18}$$

where k_i represents a weighting factor.

5.2. Identification with pseudo-experimental data

Before the actual parameter identification is conducted, we must confirm that nearly exact parameters can be determined with pseudo-experimental data where the model responses and equations are not subject to errors, to satisfy condition (i) described in Section 3.1. This section, thus, first deals with the selection of appropriate experimental data sets and then the performance of the proposed method.

The most common experimental test involves completely reversed cycling between constant strain limits. Figures 13 and 14 illustrate the time history of the test and its hysteresis loops up to five cycles. A strain amplitude is selected and an axial test specimen is loaded until the tensile

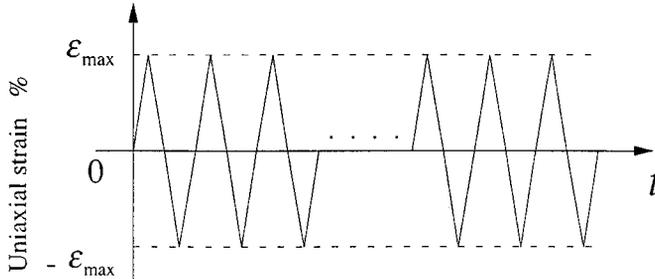


Figure 13. Common cyclic loading test

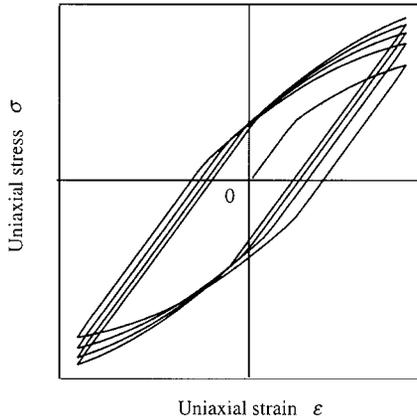


Figure 14. Hysteresis loops by the cyclic loading test

strain reaches a value of ε_{\max} . Then the direction of loading is reversed until the strain reaches ε_{\min} .

The design of a suitable set of experiments and the performance of the proposed technique were investigated stepwise through the following three test cases:

Case I: Tensile behaviour ($m = m_T$).

Case II: I + cyclic hysteresis behaviour ($m = m_T + m_C$).

Case III: II with different strain rates ($m = m_R(m_T + m_C)$).

m is the total number of measurement data and m_T , m_C and m_R are the numbers of measurement data of the tensile behaviour, cyclic hysteresis behaviour and the number of strain rates tested.

Table II lists the number of cycles, strain range, strain rate and the number of stress-strain data used in the tests. The data of the tensile behaviour ($m_T = 9$) were obtained at every 0.004 per cent strain increment, whilst the data of the cyclic hysteresis behaviour ($m_C = 10$) were obtained at $\varepsilon = \varepsilon_{\max}$ for all cycles. In Case III, cyclic hysteresis behaviours with two different strain rates were used ($m_R = 2$). These stress-strain data to be used as measurement data were computed from the parameter set $\mathbf{x}^T = [50, 3, 5000, 100, 300, 0.6, 50]$. This parameter set, therefore, must be identified from the stress-strain data. Internal parameters of the proposed technique used for the parameter identification were again those in Table I. All weighting factors k_i in the objective function were set to one. Note here that the initial parameter set for the proposed technique is not described as the initial parameter set has little influence on the performance of the algorithm, by the fact that the algorithm starts with many randomly selected parameter sets.

The objective function values of Cases I–III vs. generations are shown in Figure 15. It can be first seen that the value of the objective function successfully converged close to zero for all the cases. Material parameters in all the cases, after the objective function value became less than 0.1, are listed in Table III in comparison to the exact solution.

Figure 16 shows the curves of the tensile behaviour and the tenth cyclic hysteresis behaviour both created from the parameter set identified. The points of the tensile behaviour in the figure, termed reference points, were used to find the parameter set and the points of the tenth cyclic loading behaviour, all derived from the exact solution, are also shown as checking data. Clearly, the checking data have some distance from the tenth cyclic curve although the first tensile curve coincides with the reference data. Table III shows that only values of H and D are similar to the exact solution. As we know that the parameter set used to create the pseudo-experimental data makes the objective function value zero, the fact that the resultant objective function value with a different parameter set is close to zero indicates that there is more than one solution. Figure 17 then shows the result of Case II. The curve created is well along the reference points of both the tensile and cyclic loading behaviours. However, Table III indicates that parameters K and n are not similar to the exact solution, again indicating that more data are necessary to find the expected solution. Figure 18 shows the results of Case III. Providing different strain rates, the parameter set identified finally almost coincides with the exact solution as shown in Table III.

Table II. Parameters of uniqueness tests

	ε_{\max} (%)	$ \dot{\varepsilon} $ (s^{-1})	Material behaviour	m	m_T	m_C
Case I	0.36	8.0×10^{-3}	Tensile	9	9	0
Case II	0.36	8.0×10^{-3}	Tensile + cyclic loading	19	9	10
Case III	0.36	8.0×10^{-3}	Tensile + cyclic loading	19	9	10
	0.36	8.0×10^{-1}	Tensile + cyclic loading	19	9	10

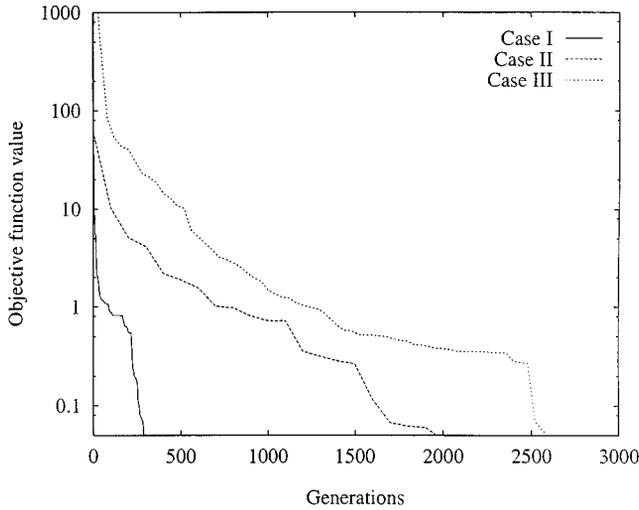


Figure 15. Objective function value vs. generations

Table III. Parameters identified in the uniqueness test

	<i>K</i>	<i>n</i>	<i>H</i>	<i>D</i>	<i>h</i>	<i>R</i> ₀	<i>d</i>
Solution	50	3	5000	100	300	50	0.6
Case I	98.3	2.46	4729	89.8	229.9	38.11	1.54
Case II	98.8	1.83	5196	105.0	293.5	52.58	0.52
Case III	49.2	2.97	5002	101.8	311.2	50.7	0.69

Bold parameters are those close to the exact solution

With the assumption that such sets of data are sufficient to determine the appropriate parameter set, the proposed technique was used to identify the parameters from ten different data sets, which were created randomly along the same curves. Resultant parameters identified had only an average of ± 0.73 per cent error at most from the exact solutions in all the cases. Although more numerical studies may be necessary, the results obtained here at least indicate that the proposed technique can find solutions almost identical to the exact solution, provided there are appropriate pseudo-experimental data.

5.3. Identification with actual experimental data

In this section, the actual experimental data of $2\frac{1}{4}$ Cr–1Mo steel under a temperature of 673 K, obtained from a benchmark project by the Society of Material Science, Japan,⁴⁰ were used to investigate the capability of the proposed method to satisfy the requirement of (ii) in Section 3.1. In conformity with the selection of data discussed in the previous section, experimental stress–strain data used for the parameter identification included information on material behaviours with different cycles and strain rates, and they are shown in Figure 19.

Parameters were identified with two other methods for comparison; one is a conventional stepwise technique⁸ and the other is a technique where a gradient-based optimization method⁴¹

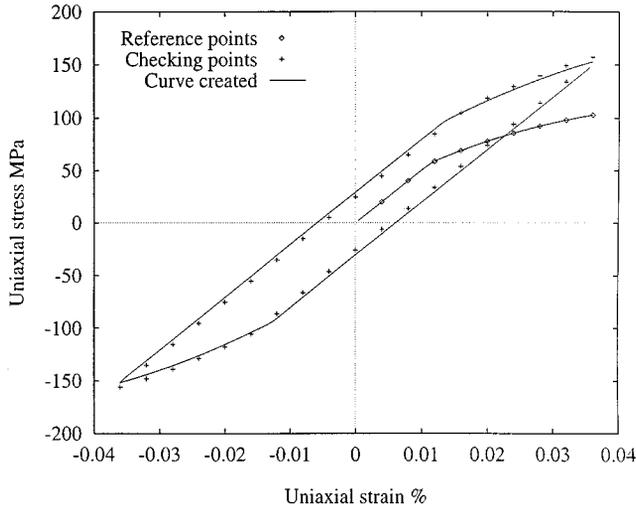


Figure 16. Comparison between reference points and estimated curve for Case I

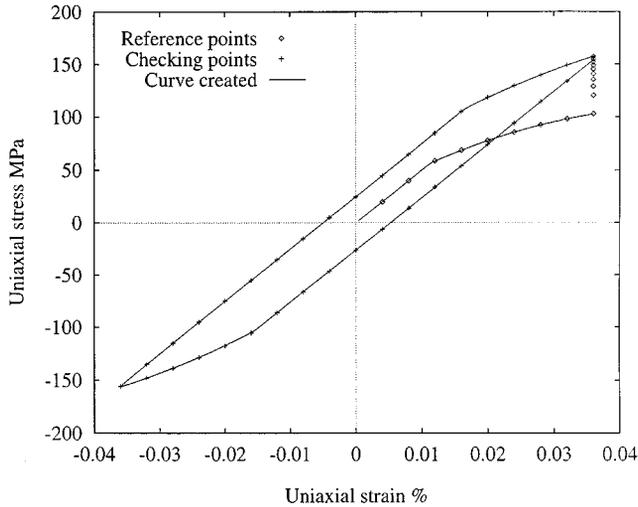


Figure 17. Comparison between reference points and estimated curve for Case II

was used to minimize the objective function (18). In Hishida's technique, parameters K , n , H and D are first determined by means of the least square method after the constitutive law is simplified by letting the yield stress R be constant. Parameters h , d and R_0 are then determined in the second step.

Table IV lists the resultant objective function value by each technique after the solution has converged enough together with the values of their initial parameter set and the solution vectors. The solution vector obtained by the proposed technique was $\mathbf{x}^T = [214, 8.96, 31500, 1690, 157, 2.48, 37.5]$. As shown in the table, the objective function value with the gradient-based technique

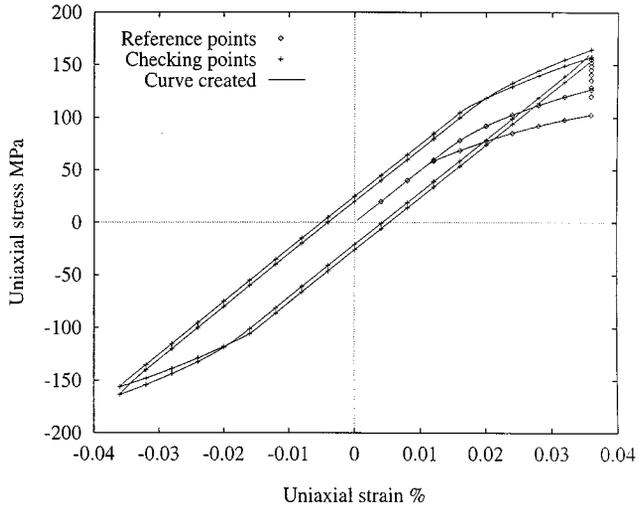


Figure 18. Comparison between reference points and estimated curve for Case III

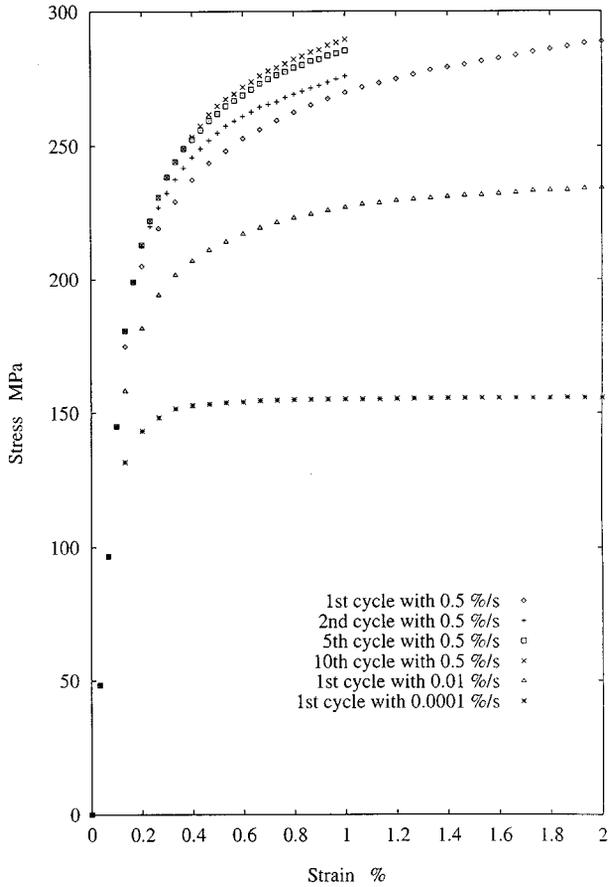


Figure 19. Experimental data of 2 1/4 Cr-1 Mo steel

Table IV. Parameters identified under measurement errors

	Initial parameter set							Objective function value
	K	n	H	D	d	h	R_0	
Proposed method								2.13×10^3
Gradient-based technique	200	5	20000	300	100	5	0	2.13×10^3
Stepwise method	50	5	20000	300	100	5	0	∞
Stepwise method	50	5	20000	300	100	5	0	3.22×10^3

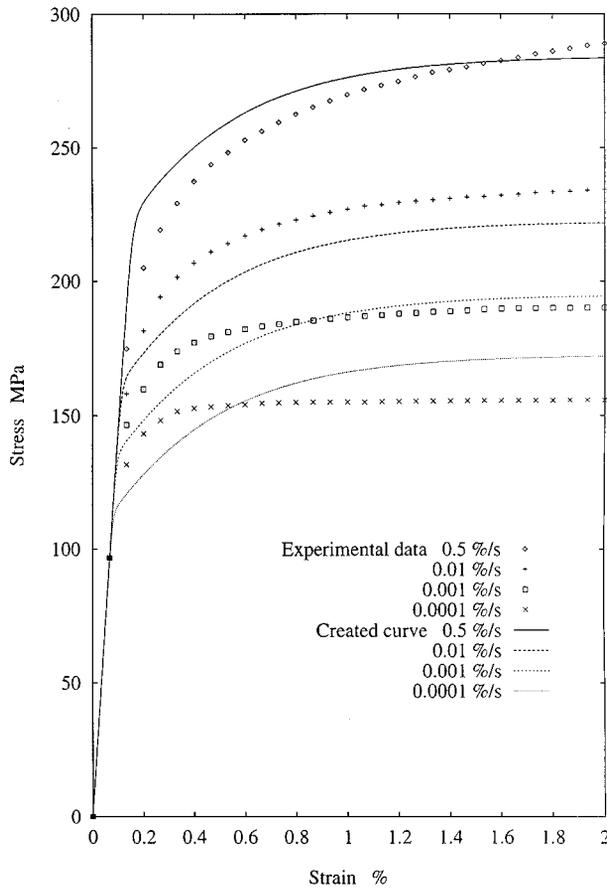


Figure 20. Computed material curves vs. experimental data

from the initial parameter set $\mathbf{x}^T = [200, 5, 20000, 300, 100, 5, 0]$ was almost identical to that with the proposed method, finding the same solution vector. In addition, the computation time with the gradient-based method was 1231 s, compared to 1847 s with the proposed technique when SPARKStation 10 was used. However, the cost functional by the gradient-based technique diverged when the initial parameter set was $\mathbf{x}^T = [50, 5, 20000, 300, 100, 5, 0]$ though only the

value of K differed. Clearly this indicates that the successful performance of the technique largely depends on the initial parameter set chosen. The stepwise technique could successfully find a stable parameter set even when a different initial parameter set was selected. However, the technique created a larger cost functional value than did the proposed method.

Curves with different strain rates, created from the proposed method, are shown in Figure 20 together with the experimental data used for the parameter identification. Experimental data with strain rate 0.001 per cent/s, which were not used for the identification, and their corresponding curve created are also shown in the figure to show the appropriateness of the parameter set identified. First, we can see that there exist model errors to some degree, which cannot be removed unless we change the model used. However, the curves created are reasonably close to the experimental data, indicating that the proposed technique is adequate for finding a parameter set which describes good approximate material behaviours.

6. CONCLUSIONS

A method for identifying the parameter set of inelastic constitutive equations, which is based on an EA, has been proposed. The algorithm, proposed by the authors, was formulated specifically for optimization with continuous search space, and it was found that the algorithm could search the optimal solution of test functions more efficiently in performance, computation time and memory usage than the canonical GA.

The proposed technique was first used to identify the parameter set of Chaboche's model under the uniaxial loading and stationary temperature conditions from the pseudo-experimental data created by the same Chaboche's model. Numerical examples show that solutions close to the exact solutions can be determined by the technique, provided there are appropriate data.

The proposed method was then tested for the parameter identification of Chaboche's model with the actual experimental data of $2\frac{1}{4}$ Cr-1Mo steel under a temperature of 673 K, and its results were compared to those by two other methods. The results of the comparison show that the technique is more accurate than methods determining parameters stepwise and is more reliable in obtaining a convergent solution, indicating the appropriateness of the technique for the parameter identification of inelastic constitutive equations.

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