

Multi-objective Parameter Identification of Unified Material Models

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1. INTRODUCTION

Mechanical behaviours of materials differ from material to material, and a number of constitutive models have been developed to accurately describe each behaviour [1]. Out of them, models based on the unified theory, combining the plastic and viscous strains can handle a variety of behaviours such as cyclic, stress relaxation and creep behaviours [2]. These models are themselves very complex in order to handle a variety of material behaviours, and, because of the variety, it is difficult to determine the best material parameters. For instance, one may adjust the parameters to a specific experiment at the expense of other materials. Another may want to have the parameters which can fit to all the experiments. In such cases, techniques which find the best fit parameters for one type of experiment, can no longer be used. Otherwise, one may make a single-objective objective function by assigning a weight to each experiment and solve the problem with an optimisation method. The solution of this approach however depends upon what weighting factor is chosen for each experiment.

In this paper, a parameter identification technique using a multi-objective optimisation method is presented. The use of the multi-objective method eliminates weighting factors from the formulation by deriving multiple solutions rather than deriving a single solution [3]. In the next section, multi-objective formulation of the constitutive parameter identification is presented. The third section deals with the multi-objective optimisation method, namely multi-objective continuous evolutionary algorithms (MCEAs), which stems continuous evolutionary algorithms (CEAs) previously developed by the authors [4]. Numerical examples and conclusions are described in the following sections.

2. FORMULATION

Inelastic constitutive equations describe stress-strain relationship of material behaviours in inelastic range and are, in a natural sense, given by

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon}; \mathbf{x}) \quad (1a)$$

for strain control where the strain for all t is defined *a priori* and

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}(\boldsymbol{\sigma}; \mathbf{x}) \quad (1b)$$

for stress control, which defines stress with respect to time beforehand. In the equations, $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ and \mathbf{x} represent the stress, strain and parameters respectively.

Such constitutive models can be typically classified into two types: models having only observable variables, i.e., stress and strain, and those having variables describing material internal behaviours as well as observable variables. One simple model for the former is Ramberg-Osgood model [5], which is given by

$$\boldsymbol{\varepsilon} = \frac{\boldsymbol{\sigma}}{E} + a \left(\frac{\boldsymbol{\sigma}}{b} \right)^n, \quad (2)$$

where a , b and n are parameters to be identified. A typical example of the latter may be Chaboche's model [2], based on the unified theory:

$$\boldsymbol{\sigma} = E(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{vp}), \quad (3a)$$

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \left\langle \frac{|\boldsymbol{\sigma} - \boldsymbol{\chi}| - R}{K} \right\rangle^n \text{sgn}(\boldsymbol{\sigma} - \boldsymbol{\chi}), \quad (3b)$$

$$\dot{\boldsymbol{\varepsilon}} = H\dot{\boldsymbol{\varepsilon}}^{vp} - D\boldsymbol{\chi}|\dot{\boldsymbol{\varepsilon}}^{vp}|, \quad (3c)$$

$$\dot{\mathbf{R}} = h|\dot{\boldsymbol{\varepsilon}}^{vp}| - dR|\dot{\boldsymbol{\varepsilon}}^{vp}|, \quad (3d)$$

where $\boldsymbol{\chi}$ and R represent material internal variables, describing the yield stress and the drag stress respectively. $\boldsymbol{\varepsilon}^{vp}$ and $\mathbf{x}=[K, n, H, D, h, d, E]$ are viscoplastic strain and parameters to be determined respectively.

In order for parameter identification, minimising a least square criterion has been most widely used. In this approach, optimisation techniques are used to find parameters by adjusting them until the measured data match the corresponding data computed from the parameter set in the least square fashion. Suppose that m experiments were conducted, each corresponding to cyclic, stress relaxation and creep behaviours and having n_j experimental data, $j=1, \dots, m$. As the measured data are stress-strain data $[\boldsymbol{\varepsilon}_i^*, \boldsymbol{\sigma}_i^*]$, the identification problem is traditionally formulated as:

$$\min_{\mathbf{x}} \sum_{j=1}^m k_j \sum_{i=1}^{m_j} \|\boldsymbol{\sigma}_i^* - \boldsymbol{\sigma}(\boldsymbol{\varepsilon}_i^*; \mathbf{x})\|^2 \quad (4a)$$

for strain control such as stress relaxation and

$$\min_{\mathbf{x}} \sum_{j=1}^m k_j \sum_{i=1}^{m_j} \|\boldsymbol{\varepsilon}_i^* - \boldsymbol{\varepsilon}(\boldsymbol{\sigma}_i^*; \mathbf{x})\|^2 \quad (4b)$$

for stress control such as creep, subject to the parameter space constraints:

$$\mathbf{x}_{\min} \leq \mathbf{x} \leq \mathbf{x}_{\max}, \quad (5)$$

where k_j is a scaling factor. The problem of this formulation is the dependence of the solution upon the scaling factors, which is inevitable for single-objective optimisation.

The formulation proposed in the paper, on the other hand, is given by

$$\min_{\mathbf{x}} \left[\sum_{i=1}^{m_1} \|\boldsymbol{\sigma}_i^* - \boldsymbol{\sigma}(\boldsymbol{\varepsilon}_i^*; \mathbf{x})\|^2, \dots, \sum_{i=1}^{m_m} \|\boldsymbol{\sigma}_i^* - \boldsymbol{\sigma}(\boldsymbol{\varepsilon}_i^*; \mathbf{x})\|^2 \right] \quad (6a)$$

for strain control and

$$\min_{\mathbf{x}} \left[\sum_{i=1}^{m_1} \|\boldsymbol{\varepsilon}_i^* - \boldsymbol{\varepsilon}(\boldsymbol{\sigma}_i^*; \mathbf{x})\|^2, \dots, \sum_{i=1}^{m_m} \|\boldsymbol{\varepsilon}_i^* - \boldsymbol{\varepsilon}(\boldsymbol{\sigma}_i^*; \mathbf{x})\|^2 \right] \quad (6b)$$

for stress control. Note that both the strain and stress control material behaviours can be together used to identify a set of parameters because of the multi-objective formulation. This problem is multi-objective, so that the conventional single-objective optimisation methods cannot be applied, thereby giving rise to the necessity for a multi-objective optimisation method.

3. MCEA

3.1. Individual representation

The fundamental structure of MCEA is based on the algorithms proposed by the authors, which is efficient for problems with continuous search space. First, a population of individuals, each represented by a continuous vector, is initially (generation $t=0$) generated at random, i.e.,

$$P^t = \{\mathbf{x}_1^t, \dots, \mathbf{x}_\lambda^t\} \in (R^n)^\lambda, \quad (7)$$

where λ represent the population size of parental individuals [6]. Each vector thus represents a search point, which corresponds to the phenomenological representation of individual.

3.2. Reproduction

The definition of the recombination and mutation becomes the probabilistic distribution of the phenomenological measures accordingly. In the recombination, parental individuals breed offspring individuals by combining part of the information from the parental individuals, thereby creating new points inheriting some information from the old points. The recombination operation is then defined as

$$\begin{cases} \mathbf{x}'_{\alpha} = (1 - \mu)\mathbf{x}_{\alpha} + \mu\mathbf{x}_{\beta} \\ \mathbf{x}'_{\beta} = \mu\mathbf{x}_{\alpha} + (1 - \mu)\mathbf{x}_{\beta} \end{cases}, \quad (8)$$

where parameter μ may be defined by the normal distribution with mean 0 and standard deviation σ :

$$\mu = N(0, \sigma^2) \quad (9)$$

or simply a uniform distribution:

$$\mu = \text{rand}(\mu_{\min}, \mu_{\max}). \quad (10)$$

The mutation can also be achieved simply by implementing

$$\mathbf{x}'' = \text{rand}(\mathbf{x}_{\min}, \mathbf{x}_{\max}). \quad (11)$$

with a small possibility. Note that the mutation may not be necessary for parameter μ with normal distribution since it can allow individuals to alter largely with small possibility, when the coefficient μ is large.

3.3. Evaluation and selection

As the Pareto-optimal set [7] is to be found as solutions, the ranking process of individuals is composed of an elimination rule. In the rule, all the points are first concerned and the Pareto-optimal set is ranked No. 1. The points in rank No. 1 are then eliminated, and the points in No. 2 are ranked as the second Pareto-optimal set, and all the other ranked are generated stepwise in the same fashion [8]. The points in rank No. k , $G(k)$, are defined as

$$G(k) = \{\mathbf{x}_i \mid \text{rank}(\mathbf{x}_i) = k, \forall i \in \{1, \dots, n\}\} \quad (12)$$

for further convenience.

The evaluation process starts with finding the best and worst objective function value of each point:

$$f_{\text{best}j} = \min \{f_j(\mathbf{x}_i) \mid \forall i \in \{1, \dots, n\}\}, \quad (13a)$$

and

$$f_{\text{worst}j} = \max \{f_j(\mathbf{x}_i) \mid \forall i \in \{1, \dots, n\}\}. \quad (13b)$$

If we temporarily define the fitness as

$$\Phi'_j(\mathbf{x}_i) = \frac{f_{\text{worst}j} - f_j(\mathbf{x}_i)}{f_{\text{worst}j} - f_{\text{best}j}}, \quad (14)$$

we can get the normalised conditions:

$$0 \leq \Phi'_j(\mathbf{x}_i) \leq 1, \quad (15)$$

and this allows us to treat the fitness of each function with the same scale. The fitness of points with the same rank has to be the same, and the true fitness of each objective function is thus defined as:

$$\Phi_j(\mathbf{x}_i) \equiv \Phi_j^{G(k)}(\mathbf{x}_i) = \max \{ \Phi'_j(\mathbf{x}_i) \mid \mathbf{x}_i \in G(k) \} \quad (16)$$

The fitness of each individual can be conclusively calculated as:

$$\Phi(\mathbf{x}_i) = \sum_{j=1}^m \Phi_j(\mathbf{x}_i), \quad (17)$$

which has the range

$$0 \leq \Phi'_j(\mathbf{x}_i) \leq m. \quad (18)$$

The selection operator favourably selects individuals of higher fitness to produce more often than those of lower fitness. As $\Phi(\mathbf{x}_i) \geq 0$ is satisfied by this equation, the proportional selection [8], which is reported to be faster in convergence than the other popular selection of the ranking selection, can be directly used in the proposed algorithm. In this selection, the reproduction probabilities of individuals are given by their relative fitness:

$$P_s(\mathbf{x}_i) = \frac{\Phi(\mathbf{x}_i)}{\sum_{j=1}^{\lambda} \Phi(\mathbf{x}_j)}. \quad (19)$$

These reproductive operations form one generation of the evolutionary process, which corresponds to one iteration in the algorithm, and the iteration is repeated until a given terminal criterion is satisfied.

4. NUMERICAL EXAMPLES

Material constants of Chaboche's model were identified simultaneously from experimental data of cyclic and creep tests by the proposed method. The results of the identification show that the Pareto-optimal solutions obtained through the identification correlate well with both the cyclic and creep tests. The proposed technique also allowed one to easily find the final solution in the search space.

5. CONCLUSIONS

Multi-objective constitutive parameter identification and an optimisation method to solve this class of problems have been presented. Numerical examples have demonstrated the effectiveness of the proposed technique. The results of the examples have not been included in this abstract due to the lack of space, but will be presented in the final version of the paper.

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